

Find the center of mass of the region between the curves $y = 11x^2$ and $y = 12x - x^2$.

SCORE: ____ / 14 PTS

$$11x^2 = 12x - x^2$$

$$12x^2 - 12x = 0$$

$$12x(x-1) = 0$$

$$x = 0, 1 \quad \textcircled{1}$$



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$$\int_0^1 (12x - x^2 - 11x^2) dx = \int_0^1 (12x - 12x^2) dx = \left(6x^2 - 4x^3 \right) \Big|_0^1 = 2 \quad \textcircled{1}$$

$$\int_0^1 x(12x - 12x^2) dx = \int_0^1 (12x^2 - 12x^3) dx = \left(4x^3 - 3x^4 \right) \Big|_0^1 = 1 \quad \textcircled{1}$$

$$\begin{aligned} \frac{1}{2} \int_0^1 ((12x - x^2)^2 - (11x^2)^2) dx &= \frac{1}{2} \int_0^1 (144x^2 - 24x^3 - 120x^4) dx \quad \textcircled{1/2} \\ &= \frac{1}{2} (48x^3 - 6x^4 - 24x^5) \Big|_0^1 = 9 \quad \textcircled{1} \end{aligned}$$

$$\frac{1}{2}(1, 9) = \left(\frac{1}{2}, \frac{9}{2} \right) \quad \textcircled{2}$$

Find the length of the parametric curve $x = 1 + 4t^{\frac{3}{2}}$ for $1 \leq t \leq e$.
 $y = \frac{3}{4}t^3 - 4\ln t$

SCORE: ____ / 6 PTS

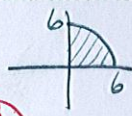
$$\begin{aligned} & \int_1^e \sqrt{(6t^{\frac{1}{2}})^2 + \left(\frac{9}{4}t^2 - \frac{4}{t}\right)^2} dt \quad \textcircled{2} \\ &= \int_1^e \sqrt{36t + \frac{81}{16}t^4 - 18t + \frac{16}{t^2}} dt \\ &= \int_1^e \sqrt{\frac{81}{16}t^4 + 18t + \frac{16}{t^2}} dt \quad \textcircled{1} \\ &= \int_1^e \left(\frac{9}{4}t^2 + \frac{4}{t}\right) dt \quad \textcircled{1} \\ &= \left(\frac{3}{4}t^3 + 4\ln t\right) \Big|_1^e = \frac{3}{4}e^3 + 4 - \frac{3}{4} = \frac{3}{4}e^3 + \frac{13}{4} \quad \textcircled{1} \end{aligned}$$

The time for a tax refund check to be deposited after it has been received is a continuous random variable X

SCORE: ____ / 10 PTS

(in units of days) with probability density function $f(x) = \begin{cases} k\sqrt{36-x^2}, & x \in [0, 6] \\ 0, & x \notin [0, 6] \end{cases}$ for some constant k .

Find the mean (average) time before a tax refund check is deposited.

$$\int_0^6 k \sqrt{36-x^2} dx = 1$$

$$k \left(\frac{1}{4} \cdot \pi (6)^2 \right) = 1$$
$$k = \frac{1}{9\pi}$$

$$\int_0^6 \frac{1}{9\pi} x \sqrt{36-x^2} dx$$
$$= \int_{36}^0 \frac{1}{9\pi} \left(-\frac{1}{2} \sqrt{u} \right) du$$
$$= -\frac{1}{18\pi} \frac{2}{3} u^{\frac{3}{2}} \Big|_{36}^0$$
$$= \frac{1}{27\pi} \cdot 36^{\frac{3}{2}}$$
$$= \frac{1}{27\pi} 6^3$$
$$= \frac{8}{\pi} \text{ DAYS}$$

$$u = 36 - x^2$$
$$du = -2x dx$$
$$-\frac{1}{2} du = x dx$$
$$x=6 \rightarrow u=0$$
$$x=0 \rightarrow u=36$$